UNIT-1

(Lecture-7)

Realization of Digital Systems:
FIR Digital Filter Structures

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FIR Digital Filter Structures

- A causal FIR filter of order N is characterized by a transfer function $H(z)$ given by
$$H(z) = \sum_{n=0}^{N} h[n] z^{-n}$$
which is a polynomial in z^{-1}
- In the time-domain the input-output relation of the above FIR filter is given by
$$y[n] = \sum_{k=0}^{N} h[k] x[n-k]$$

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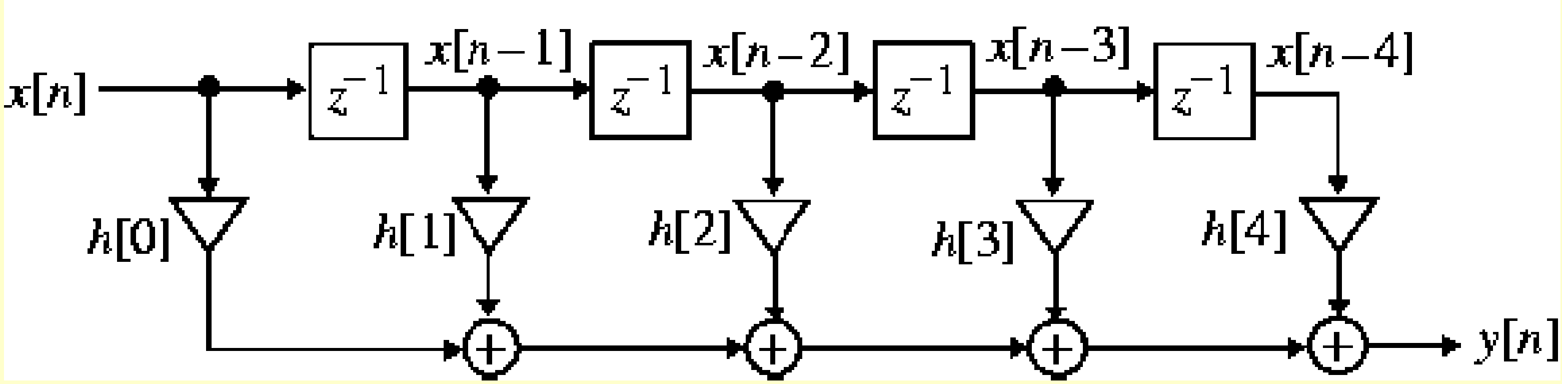
Direct Form FIR Digital Filter Structures

- An FIR filter of order N is characterized by $N+1$ coefficients and, in general, require $N+1$ multipliers and N two-input adders
- Structures in which the multiplier coefficients are precisely the coefficients of the transfer function are called **direct form** structures

Direct Form FIR Digital Filter Structures

- A direct form realization of an FIR filter can be readily developed from the convolution sum description for $N = 4$



Direct Form FIR Digital Filter Structures

- An analysis of this structure yields

$$y[n] = h[0]x[n] + h[1]x[n-1] + h[2]x[n-2] + h[3]x[n-3] + h[4]x[n-4]$$

  which is precisely of the form of the convolution sum description

- The direct form structure shown on the previous slide is also known as a **tapped delay line** or a **transversal filter**

Cascade Form FIR Digital Filter Structures

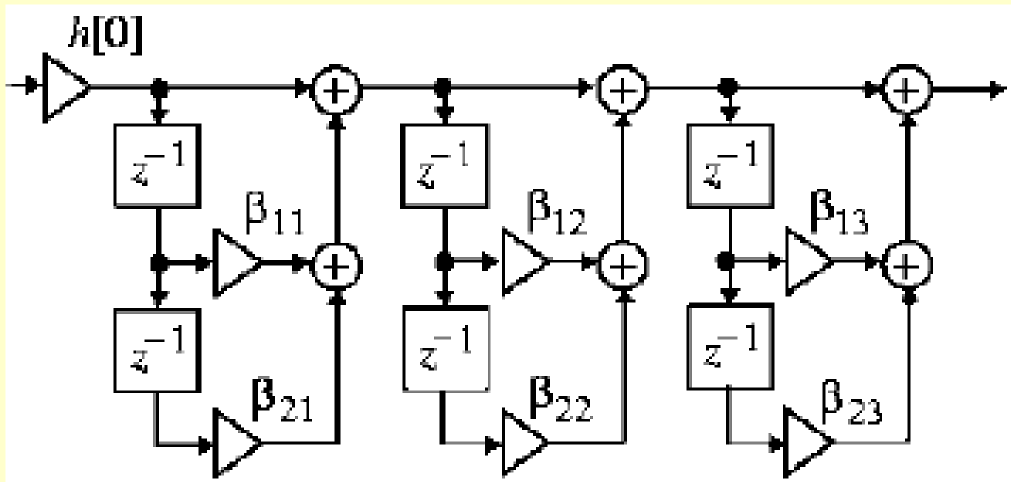
- A higher-order FIR transfer function can also be realized as a cascade of second-order FIR sections and possibly a first-order section
- To this end we express $H(z)$ as

$$H(z) = h[0] \cdot \prod_{k=1}^{K} (1 + \beta_{1k} z^{-1} + \beta_{2k} z^{-2})$$

where $K = \frac{N}{2}$ if N is even, and $K = \frac{N+1}{2}$ if N is odd, with $\beta_{2K} = 0$

Cascade Form FIR Digital Filter Structures

- A cascade realization for $N = 6$ is shown below



- Each second-order section in the above structure can also be realized in the transposed direct form

Linear-Phase FIR Structures

- The symmetry (or antisymmetry) property of a linear-phase FIR filter can be exploited to reduce the number of multipliers into almost half of that in the direct form implementations
- Consider a length-7 **Type 1** FIR transfer function with a symmetric impulse response:

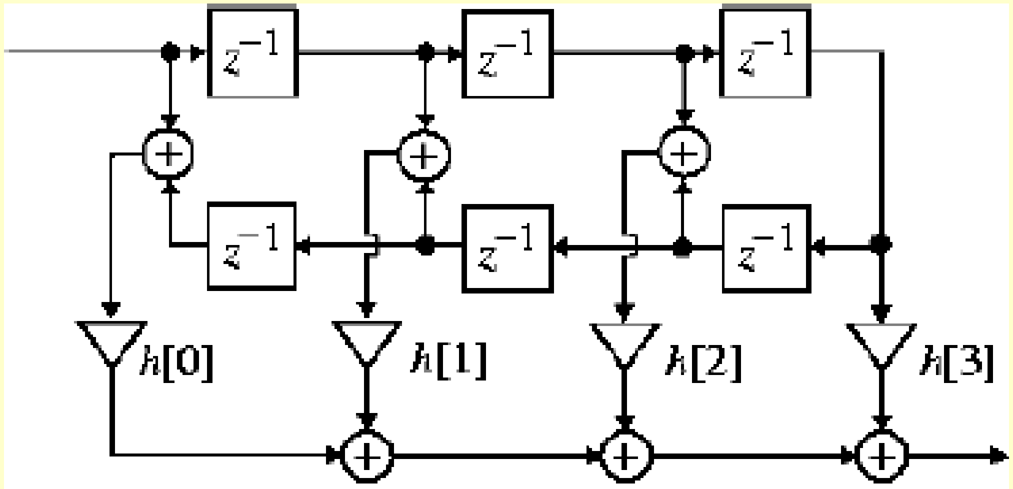
$$H(z) = h[0] + h[1]z^{-1} + h[2]z^{-2} + h[3]z^{-3}$$
$$+ h[2]z^{-4} + h[1]z^{-5} + h[0]z^{-6}$$

Linear-Phase FIR Structures

- Rewriting $H(z)$ in the form

$$H(z) = h[0](1 + z^{-6}) + h[1](z^{-1} + z^{-5}) + h[2](z^{-2} + z^{-4}) + h[3]z^{-3}$$

we obtain the realization shown below



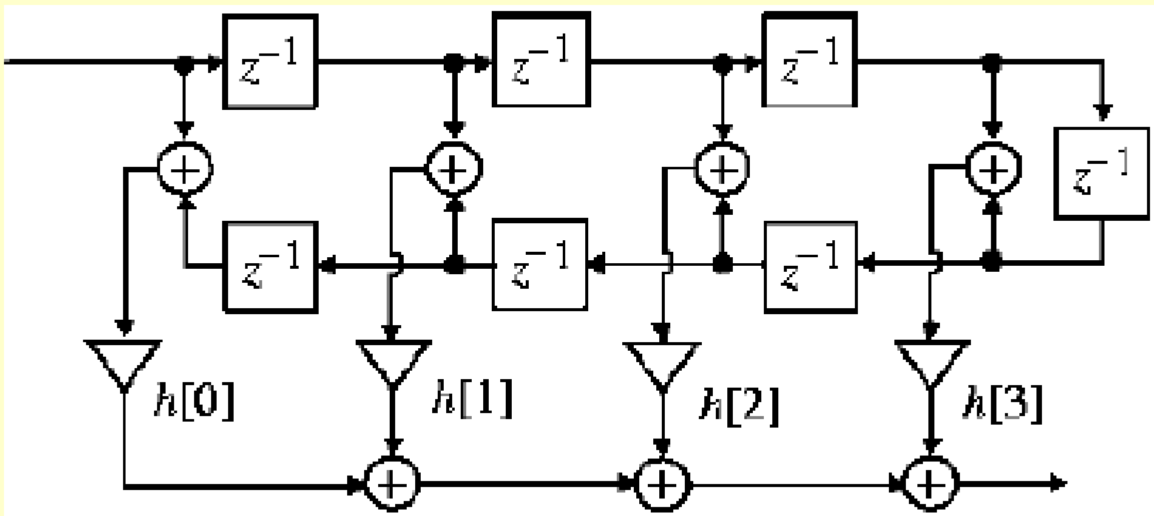
Linear-Phase FIR Structures

- Note: The Type 1 linear-phase structure for a length-7 FIR filter requires 4 multipliers, whereas a direct form realization requires 7 multipliers
- A similar decomposition can be applied to a Type 2 FIR transfer function
- For example, a length-8 **Type 2** FIR transfer function can be expressed as

Linear-Phase FIR Structures

$$H(z) = h[0](1 + z^{-7}) + h[1](z^{-1} + z^{-6})$$
$$+ h[2](z^{-2} + z^{-5}) + h[3](z^{-3} + z^{-4})$$

leading to the realization shown below



Linear-Phase FIR Structures

- Note: The Type 2 linear-phase structure for a length-8 FIR filter requires 4 multipliers, whereas a direct form realization requires 8 multipliers
- Similar savings occurs in the realization of Type 3 and Type 4 linear-phase FIR filters with antisymmetric impulse responses